# Estuarine Salinity Structure

tidally-averaged

8/7/2019

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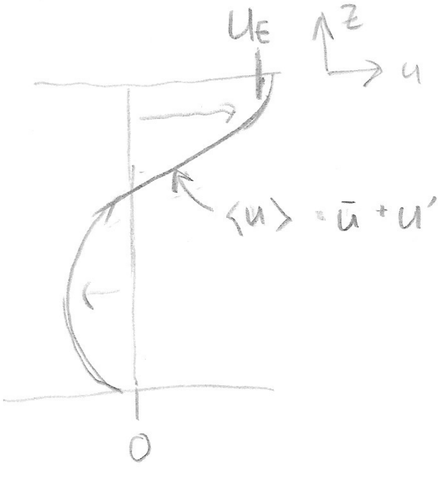
Recall: the exchange flow looks like:



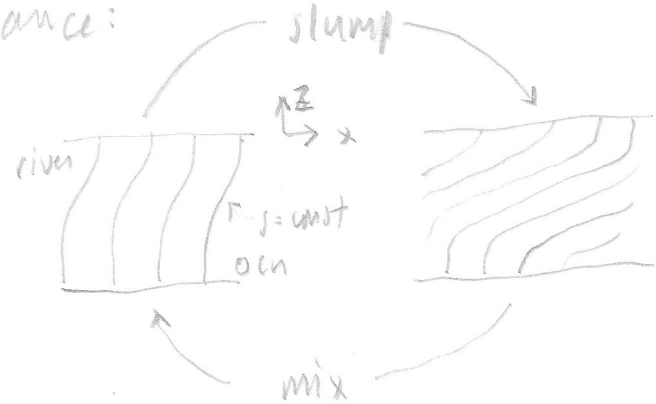

where $u_E = \frac{g\beta \bar{s}_x H}{48} \frac{H^2}{A}$

represents the balance between pressure gradient (increase exchange) and friction (slow exchange)

The stratification $s'(z)$ is mainly controlled by a similar local balance:

$$\boxed{s'_t = -u'\bar{s}_x + K s'_{zz}}$$

slump mix




Find steady solution: $s'_{zz} = \frac{\bar{s}_x}{K} u'$ with $s'_z = 0$ at $z = -H, 0$

cubic

(no flux through bottom or top)

Result

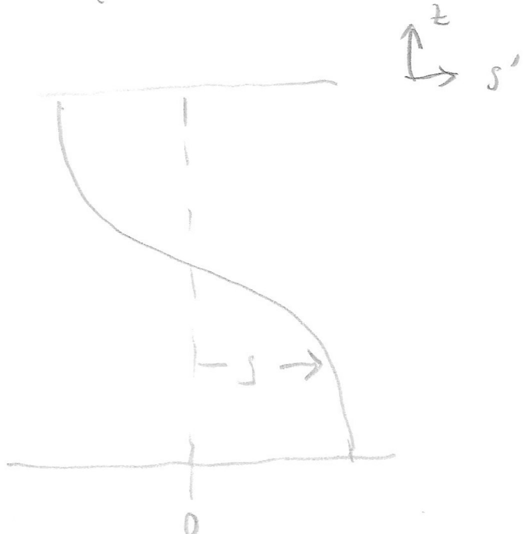
$s' = \frac{H^2}{K} \bar{s}_x (\bar{u} P_1 + u_E P_2)$

where $P_1 = -\frac{7}{120} + \frac{1}{4}\zeta^2 - \frac{1}{8}\zeta^4$

$P_2 = -\frac{1}{12} + \frac{1}{2}\zeta^2 - \frac{3}{4}\zeta^4 - \frac{2}{5}\zeta^5$

and $\zeta = \frac{z}{H}$

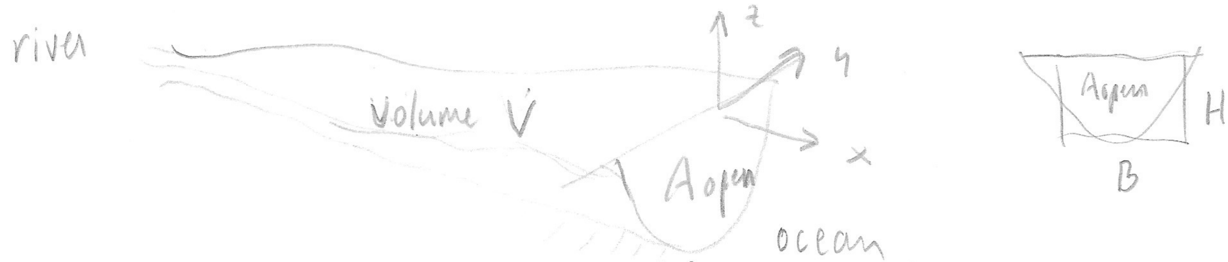
quintic



and recall $[u'] = U_E = \frac{g\beta \bar{s}_x H}{48} \frac{H^2}{K} \sim \bar{s}_x$ (assume $A = K$)

so $[s'] = \frac{H^2}{K} \bar{s}_x U_E \sim \bar{s}_x^{\,2}$ so stratification is very sensitive to $\bar{s}_x$

Next consider the volume integrated salt balance



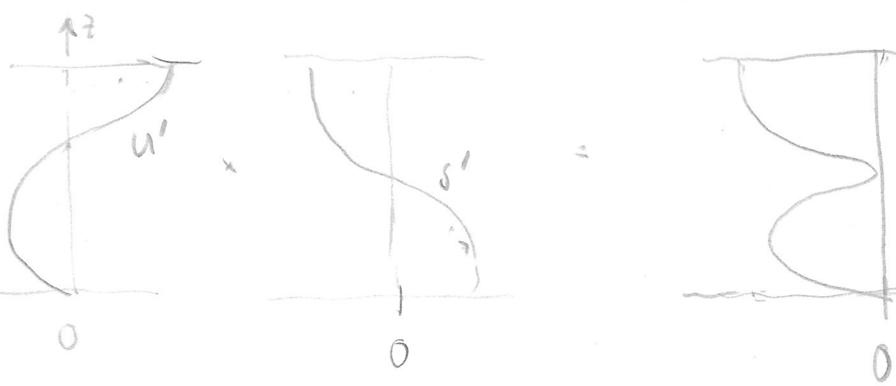

$$\frac{d}{dt}\int_V s\, dV = -\int_{A_{open}} us\, dA \cong -\int_{-B/2}^{B/2}\int_{-H}^{0} (\bar{u}+u')(\bar{s}+s')\, dz\, dy$$

$$\cong HB\left(\underset{①}{-\bar{u}\bar{s}} \; \underset{②}{-\overline{u's'}}\right)$$

Most important terms are

① River flow $\times$ $\bar{s}$ removes salt $(-\bar{u}\bar{s})$

② exchange flow $\times$ stratification brings salt in $(-\overline{u's'})$




$u's'$ always negative

$\Rightarrow -\overline{u's'} > 0$

and $-\overline{u's'} \propto \bar{s}_x^{\,3}$ ☆

In steady state we can use the salt flux through the mouth

$$0 = \underset{(1)}{-\bar{u}\bar{s}} - \underset{(2)}{\overline{u's'}} \quad (*)$$

to come up with an equation for $\bar{s}(x)$.

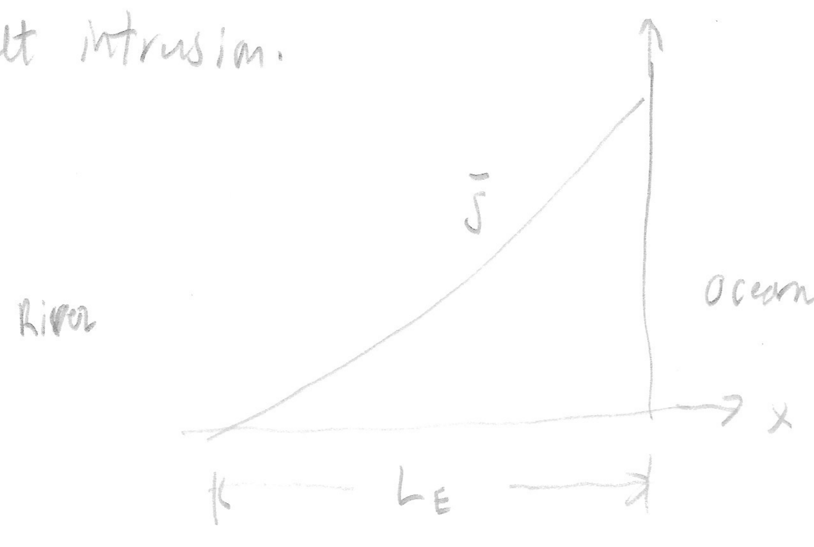
... After some algebra we can write $(*)$ as

$$(\circ) \quad \underset{(2)}{L_E^3 \Sigma_x^3} - \underset{(1)}{\Sigma} = 0 \qquad \text{where} \quad \Sigma \equiv \bar{s}/s_{ocn}$$

and $L_E = 0.024\, c \left(\frac{\bar{u}}{c}\right)^{-1/3} \frac{H^2}{K}$

and $c = \sqrt{g\beta s_{ocn} H}$ ~ 2× the fastest possible internal wave, with $\frac{\Delta\rho}{\rho_0} = \beta s_{ocn}$

And $L_E$ is the scale of the length of the salt intrusion.



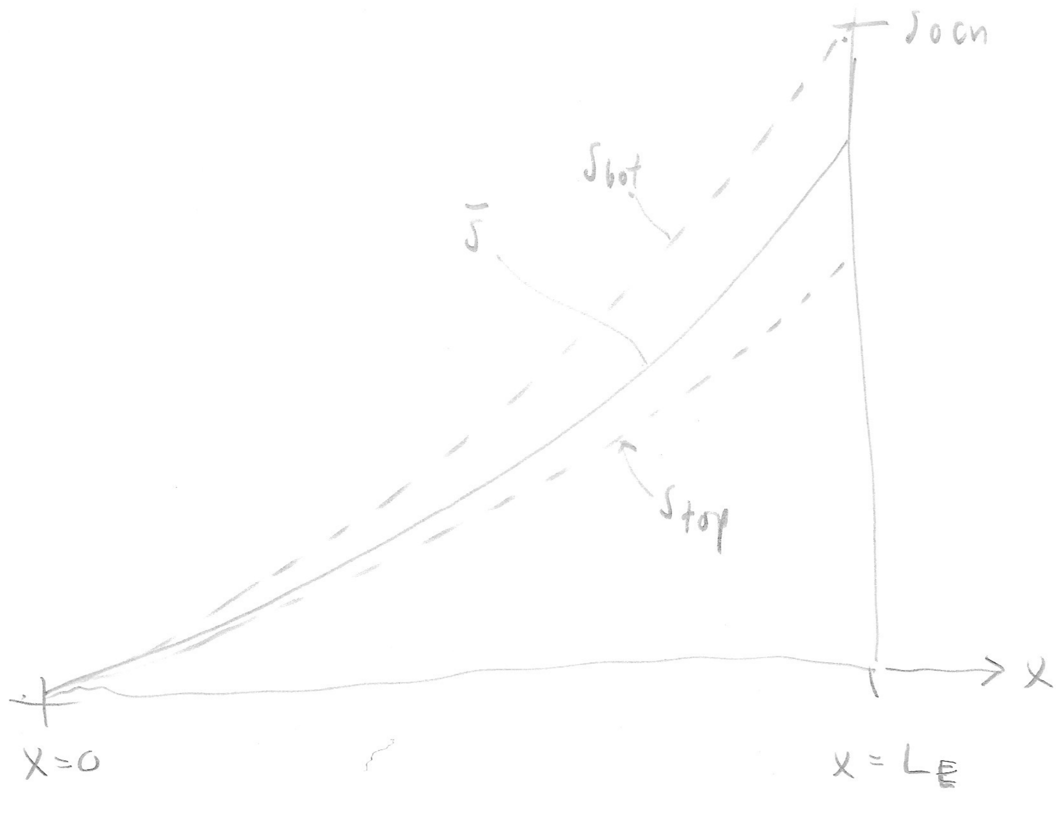
Solving (•), guess $\Sigma = \alpha x^n$

$$\Sigma_x = n\alpha x^{n-1}$$

$$(\bullet) \quad L_E^3 n^3 \alpha^3 x^{3n-3} - \alpha x^n = 0$$

In order for the solution to work for all $x$ we require $3n-3 = n \Rightarrow n = 3/2$

$$\Rightarrow \Sigma = \alpha x^{3/2}$$

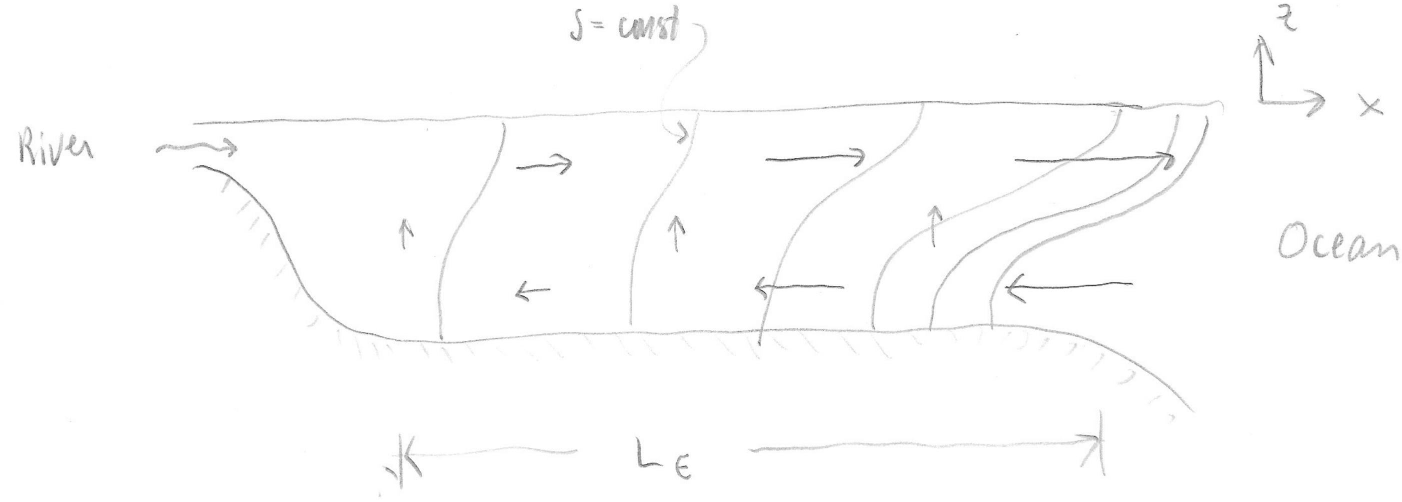



So $\bar{S} \sim x^{3/2}$

$S' \sim x$

$u' \sim x^{1/2}$

$$\bar{S}_x \sim \frac{S_{ocn}}{L_E}$$




Scaling: Recall $L_E = 0.024\, c \left(\frac{\bar{u}}{c}\right)^{-1/3} \frac{H^2}{K}$, $c = \sqrt{g\beta S_{ocn} H}$

and $\bar{S}_x \sim \frac{S_{ocn}}{L_E}$

$$U_E = \frac{g\beta \bar{S}_x H}{48} \frac{H^2}{K} \sim \frac{c^2}{48} \frac{1}{L_E} \frac{H^2}{K} \ldots$$

$$\frac{[S']}{S_{ocn}} \sim \frac{H^2}{K} \frac{\bar{S}_x}{S_{ocn}} U_E \sim \left(\frac{1}{L_E}\right)^2 \left(\frac{H^2}{K}\right)^2 c^2 \ldots$$

## Scaling

$$\boxed{L_E \sim c\left(\frac{\bar{u}}{c}\right)^{-1/3} \frac{H^2}{K}} \quad , \quad c = \sqrt{g\beta s_{ocn} H}$$

$\Rightarrow L_E$ – shorter for more mixing $\left(\sim \frac{1}{K}\right)$

– shorter for more river flow
but it is stiff $L_E \sim Q_R^{-1/3}$

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$$u_E \sim g\beta \bar{s}_x H \frac{H^2}{K} \sim \frac{g\beta s_{ocn} H}{L_E} \frac{H^2}{K} = \frac{c^2}{L_E} \frac{H^2}{K}$$

so $u_E \sim c\left(\frac{\bar{u}}{c}\right)^{\frac{1}{3}}$ – weak dependence on $Q_R$
– no dependence on $K$
(Why?)

$$\sim \boxed{\frac{u_E}{c} \sim \left(\frac{\bar{u}}{c}\right)^{\frac{1}{3}}}$$

Similarly we find

$$\frac{[S']}{S_{ocn}} \sim \frac{u_E}{L_E}\,\frac{H^2}{K} \sim \frac{C^2}{L_E^2}\left(\frac{H^2}{K}\right)^2$$

so $$\boxed{\left[\frac{S'}{S_{ocn}}\right] \sim \left(\frac{\bar{u}}{C}\right)^{2/3}}$$

again: independent of $K$